AP CALCULUS AB

Linear Approximations

1) $f(x) = x^3 + 3x$

point: $(2, 14)$

slope: $f'(x) = 3x^2 + 3$

$f'(2) = 15$

$y - 14 = 15(x-2)$

$L(x) = 14 + 15(x-2)$

$L(2.01) = 14 + 15(2.01-2)$

$L(2.01) = 14.15$

$\boxed{f(2.01) \approx 14.15}$

2) $\sqrt{24.9} + (24.9)^2$

$f(x) = \sqrt{x} + x^2$

point: $a = 25$, $(25, 630)$

slope: $f'(x) = \frac{1}{2\sqrt{x}} + 2x$

$f'(25) = \frac{1}{10} + 50 = 50.1$

$y - 630 = 50.1(x-25)$

$L(x) = 630 + 50.1(x-25)$

$L(24.9) = 630 + 50.1(24.9-25)$

$= 630 + 50.1(-0.1)$

$= 624.99$

$\boxed{f(24.9) \approx 624.99}$

3) $f(x) = \sqrt{x^2+9}$

point: $(-4, 5)$

slope: $f'(x) = x(x^2+9)^{-1/2}$

$f'(-4) = -\frac{8}{10}$

$y - 5 = -\frac{8}{10}(x+4)$

$L(x) = 5 - 0.8(x+4)$

$L(-3.9) = 5 - 0.8(-3.9+4)$

$= 5 - 0.8(0.1)$

$= 4.92$

$\boxed{f(-3.9) = 4.92}$

4) $\sqrt[4]{17} \rightarrow f(x) = x^{1/4}$

point: $(16, 2)$

slope: $f'(x) = \frac{1}{4}x^{-3/4} = \frac{1}{4(\sqrt[4]{x})^3}$

$f'(16) = \frac{1}{32}$

$y - 2 = \frac{1}{32}(x-16)$

$L(x) = 2 + \frac{1}{32}(x-16)$

$L(17) = 2 + \frac{1}{32}(1) = \frac{65}{32}$

$\boxed{f(17) \approx \frac{65}{32}}$

5) $(8.4)^{4/3} \rightarrow f(x) = x^{4/3}$

point: $(8, 16)$

slope: $f'(x) = \frac{4}{3}x^{1/3}$

$f'(8) = \frac{8}{3}$

$y - 16 = \frac{8}{3}(x-8)$

$L(x) = 16 + \frac{8}{3}(x-8)$

$L(8.4) = 16 + \frac{8}{3}(0.4)$

$= 16 + \frac{16}{15}$

$\boxed{f(8.4) = \frac{256}{15}}$

6) $y = \frac{x+1}{x-2}$

$\frac{dy}{dx} = \frac{x-2-(x+1)}{(x-2)^2} = \frac{-3}{(x-2)^2} = -3(x-2)^{-2}$

$\frac{d^2y}{dx^2} = 6(x-2)^{-3}$

7) $f(x) = \boxed{x}(x+2)^3$

$f'(x) = x\left[3(x+2)^2\right] + (x+2)^3$

$= 3x(x+2)^2 + (x+2)^3$

$= (x+2)^2\left[3x + x + 2\right]$

$= (x+2)^2(4x+2)$

$= 2(x+2)^2(2x+1)$

$f''(x) = 2(x+2)^2(2) + (2x+1)\cdot\left[4(x+2)\right]$

$= 4(x+2)^2 + 4(x+2)(2x+1) = 4(x+2)(3x+3) = \boxed{12(x+2)(x+1)}$

8) $f'(1) = 2.333$ 9) $f'(\frac{\pi}{4}) = 1$ 10) $f'(2) = 1.385$

11)

$$g(x) = \begin{cases} (x+3)^2 & x \le -2 \\ 2x+5 & -2 < x < 5 \\ (6-x)^2 & x \ge 5 \end{cases}$$

check $x = -2$

cont

I $g(-2) = 1$

II. $\lim_{x\to -2^-} (x+3)^2 = 1$

$\lim_{x\to -2^+} (2x+5) = 1$

$\lim_{x\to -2} g(x) = 1$

III. $g(-2) = \lim_{x\to -2} g(x)$

Diff

Left

$\frac{d}{dx}\left((x+3)^2\right)\Big|_{x=-2}$

$2(x+3)\Big|_{x=-2}$

2

Right

$\frac{d}{dx}(2x+5) = 2$ ✓

check $x = 5$

cont

I. $g(5) = 1$

II. $\lim_{x\to 5^-} (2x+5) = 15$

$\lim_{x\to 5^+} (6-x)^2 = 1$

$g(x)$ is not cont @ $x = 5$

$\therefore$ $g(x)$ is differentiable on $(-\infty, 5) \cup (5, \infty)$